FUNCTIONAL OPTIONS AND METHODS OF SOLVING OLYMPIC AND COMPETITIVE ISSUES

Djakaeva K.[1], Izetaeva G.[2], Narbaeva R.[3] (Republic of Uzbekistan)
Email: Djakaeva514@scientifictext.ru

[1]Djakaeva Kenje – Teacher,
ACADEMIC LYCEUM OF KARAKALPAK STATE UNIVERSITY;
[2]Izetaeva Gulbahar - PhD in pedagogical science, Assistant;
[3]Narbaeva Raziya - Assistant Teacher,
MATHEMATICAL ANALYSIS DEPARTMENT,
KARAKALPAK STATE UNIVERSITY,
NUKUS, REPUBLIC OF UZBEKISTAN

Abstract: *today, the development of science and technology is a priority for secondary schools and academic lyceums with the introduction of a system of quality education, training of qualified personnel, taking into account the needs and prospects of the labour market. This article discusses how to solve some functional equations that are encountered in academic subjects for academic lyceum and upper class olimpiads. The main purpose of solving equations studied in mathematics in secondary schools and academic lyceums is to find the numerical values of the number of unknown variables.*
Keywords: *variable value, unknown function, functional equation, continuous function, general solution.*

ФУНКЦИОНАЛЬНЫЕ УРАВНЕНИЯ И МЕТОДЫ РЕШЕНИЯ ОЛИМПИАДНЫХ И КОНКУРСНЫХ ЗАДАЧ

Джакаева К.[1], Изетаева Г.[2], Нарбаева Р.[3] (Республика Узбекистан)

[1]Джакаева Кенже – преподаватель,
академический лицей
при Каракалпакском государственном университете;
[2]Изетаева Гулбахар - PhD по педагогике, ассистент;
[3]Нарбаева Разия – ассистент,
кафедра математического анализа,
Каракалпакский государственный университет,
г. Нукус, Республика Узбекистан

Аннотация: *сегодня развитие науки и техники является приоритетом для общеобразовательных школ и академических лицеев с внедрением системы качественного образования, подготовки квалифицированных кадров с учетом потребностей и перспектив рынка труда. В этой статье обсуждается, как решить некоторые функциональные уравнения для академического лицея и старшеклассников на олимпиаде. Основная*

цель решения уравнений, изучаемых по математике в средних школах и академических лицеях, состоит в том, чтобы найти числовые значения числа неизвестных переменных.

Ключевые слова: *значение переменной, неизвестная функция, функциональное уравнение, непрерывная функция, общее решение.*

At the same time, the action Plan approved by presidential decree No. P-4947 of 7 February 2017 provides for the development of education and science as one of the priorities for the next five years. As a result of these activities, the preparation of students for Olympiads and Olympiads in mathematics in academic lyceums, training and education is a modern requirement [1-4].

There are some equations that arise in the problem of Olympics and competitions; the main purpose of solving these equations is to find unknown functions rather than numerical values of unknown variables. Some of our students have difficulty with examples related to these functional equations at Olympic games and competitions. In the school mathematics course simple functions, their properties and graphs are considered in detail. Moreover, little attention is paid to functional equations. Therefore, it would be appropriate to present them as students ' research technologies as an independent work.

The textbook of the academic Lyceum "Fundamentals of algebra and mathematical analysis" provides examples of functional equations [2], these examples are not enough for gifted students. Therefore, when teaching students the subject "Interesting mathematics and Olympiad" it is best to explain the following examples.

For example,

$$4f(x+1) = f(x) - 2x,$$

$$f(xy) = f(x) \cdot f(y),$$

$$xf(x) + f\left(\frac{1}{\alpha - x}\right) = x$$

and there are other kinds of equations, the unknown variable in such equations now consists of some functions. An example is the function of an unknown $f(x)$ variable in the above equations. Such equations are functional equations.

Equations that consist of functions rather than numbers are called functional equations.

The solution of functional equations is divided into a General solution and a particular solution. A function or class of functions that satisfy functional equations is a particular solution. A General solution is a set of functions or classes of functions that satisfy functional equations.

One of the most commonly used functional equations in school and school mathematics is in the Cauchy equation class

$$f(x+y)=f(x)+f(y) \qquad (1)$$

the equation was found. This equation represents a property called the additive property of functions. Therefore, $f(x)$ if there is an additive function, then it is a function (1) satisfies the equation. Cauchy, along with his additive equation, now more often participates in Olympiads and competitions and is used by most students to solve mathematical problems.

$$f(x+y)=f(x)\cdot f(y),$$
$$f(xy)=f(x)+f(y),$$
$$f(xy)=f(x)f(y)$$

also looks at the equations in the view. Therefore, these equations are often called Cauchy functional equations. Solutions of these equations are represented by exponential, logarithmic and level functions known in elementary mathematics.

Below we consider some mathematical questions that can be solved by solving Cauchy equations.

Example 1. To find a curve in the plane, let it be absorption units for arbitrary two points along the curve, the sum of multiplication of two other points, and the amount of points a third point, the multiplication of these two points.

Decision. To solve this problem, we restrict ourselves to finding a function whose graph is continuous and determined by the positive values of the argument.

Problem solution

$f(xy)=xf(y)+yf(x)$

as a result, the solution of the functional equation in the form. Let $g(x)=\dfrac{f(x)}{x}$

to be like this. Then this equation turns out to be one of the Cauchy equations

$g(xy)=g(x)+g(y)$

we can get such equation. $x>0$ for $g(x)$ being a continuous function, it

solution of equation will be $g(x)=C\ln x$, here C is freely unchanged. Returning to the previous function, in this way will be $f(x)=Cx\ln x$.

Example 2. $x>0$ defined for and

$$f(f(x))=xf(x) \qquad (2)$$

Decision. From the appearance of the equation there is no fixed number except zero (2) doesn't satisfy the equation. However, in possible values x from (2) will be $f(x)>0$. If there is $f(x)=y$, in this way (2) will be written like this $f(y)=xy$, from this there will be this equation $f(f(y))=f(xy)$. At the same time from (2) we will change *x* to *y* and paying attention that *y= f(x)*

$f(f(y))=y\,f(y)=f(x)f(y)$

being

$$f(xy) = f(x)f(y)$$

will have such equation. It is a function of the Cauchy function equation. Its continuous solution from scratch will be like this $f(x) = x^a$. If we take positive values in the Cauchy equation x and y and they are not interrelated, in this situation $f(x) = x^a$ will be the solution. If x and y will be connected like this $y = f(x)$, that's why the function $f(x) = x^a$ in a all (2) equations will not be satisfied.

For this purpose we can put this function into the equation, select the desired values from a.

If we put $f(x) = x^a$ in (2) equations there will be $x^{a^2} = x^{a+1}$. From this divide $a^2 = a+1$, there will be $a = \frac{1 \pm \sqrt{5}}{2}$, that's why the functions $f(x) = x^{\frac{1+\sqrt{5}}{2}}$ and $n > 1,\ f(1) = a$ will satisfy (2) equations.

Thus, the solutions of this equation

$$f(x) \equiv 0, \quad f(x) = x^{\frac{1+\sqrt{5}}{2}}, \quad f(x) = x^{\frac{1-\sqrt{5}}{2}}$$

will be this function. When solving some functional equations, it is desirable to derive equations from both sides. In this case, the equation with an unknown function in this equation becomes a kind of differential equation. This method is used in solving the differential equations of functions in solving the Cauchy function equations.

Example 3. Let

$$f(5x+1) = 25f(x), \ x \in R \qquad (3)$$

we consider the solution of a class of functions for solving derivatives of the first and second order of a functional equation. (3) that the equation has a solution that satisfies the requirements of the problem, from 2 sides of (3) can gather twice. Then will be ,

$$f'(5x+1) = 5f'(x), \qquad (4)$$

$$f''(5x+1) = f''(x) \qquad (5)$$

performing $x \to \frac{x-1}{5}$ the operation n sequentially, from (5) will get in the following sequence of equations:

$$f''(x) = f''\left(\frac{x-1}{5}\right) = f''\left(\frac{x-6}{25}\right) = \ldots = f''\left(\frac{x-(5^n-1)/4}{5^n}\right).$$

From this if we go to $n \to \infty$ limit there will be

$$f''(x) = \lim_{n\to\infty} f''\left(\frac{x-(5^n-1)/4}{5^n}\right) = f''\left(-\frac{1}{4}\right) = a$$

elementary functions of $f''(x)$ will have such kind of functions $f'(x) = ax + b$.

Now from (4) subtract $x = -\frac{1}{4}$, in this situation will be $f'\left(-\frac{1}{4}\right) = 0$, from this

$$f'\left(-\frac{1}{4}\right) = a \cdot \left(-\frac{1}{4}\right) + b = 0,$$

it means

$$a = 4b, \quad f'(x) = b(4x+1)$$

Let's continue in this way

$$f(x) = \frac{1}{4}b(4x+1)^2 + c, \quad f\left(-\frac{1}{4}\right) = 0 = c$$

So there will be $f(x) = \frac{1}{4}b(4x+1)^2$.

Well, let's look at the functional equations given in natural numbers. If the Fibonacci repetition factor for a series of numbers

$$f(n) = f(n-1) + f(n-2)$$

if we write in a form, then we will have some form of functional equation. If we solve this equation in this way $f(1) = f(2) = 1$, then we get a formula for the total number of sequences.

Now we present several ways to solve functional equations with natural argument, which have the form of the coefficient of repeatability.

One of the easiest ways to solve such a function equation is to find out the General form of the function by calculating several initial values in natural numbers, using the repetition factor of the original unknown function, and then checking the correctness of the formula found by mathematical induction. It is on the basis of these methods that a common range of arithmetic and geometric progressions can be found.

Example 4. If will be $f(1) = 3, \quad f(2) = 7$, in this situation

$$f(n) = 3f(n-1) - 2f(n-2)$$

we define a *n* formula for the sequence limit given by the repeatability factor**.**

Decision. To solve this problem, first define a few values for the function that you are searching:

$$f(3) = 3f(2) - 2f(1) = 15,$$

$$f(4) = 3f(3) - 2f(2) = 31.$$

From these two we can conclude

$$f(n) = 2^{n+1} - 1 \qquad (6)$$

Now let's see if this formula is correct for mathematical induction. First, for $n = 1$ consider $f(1) = 3 = 2^2 - 1$. Now $f(1) = 3 = 2^2 - 1$ for $f(k) = 2^{k+1} - 1$ by the coefficient of repeatability under the condition of equality

$f(n)=3f(n-1)-2f(n-2)=3(2^n-1)-2(2^{n-1}-1)=2^{n+1}-1$

we have equality. Thus, it follows that (6) is true for all positive integers.

Example 5. If there will be

$n^{f(n)-1}=(n-1)^{f(n-1)}$

in this situation find $f(n),\ n\in N$.

Decision. This is not determined by the conditions of the problem $f(0)$, so the equation $n>1$ will have a value for the natural numbers**.** If there will be $n=2$, in this situation will be $2^{f(2)-1}=1^{f(1)}$, from this there will be $f(2)=1,\ f(1)=a$, here will be $a\in R$.

Now in a row take $n=3,\ 4,\ 5$ and we will get such equations:

$$3^{f(3)-1}=2^{f(2)}\ f(3)=\log_3 2+1=\log_3 6$$
$$4^{f(4)-1}=3^{f(3)}\ 4^{f(4)-1}=3^{\log_3 6}\ f(4)=\log_4 6+1=\log_4 24$$
$$5^{f(5)-1}=4^{f(4)}\ 5^{f(5)-1}=4^{\log_4 24}\ f(5)=\log_5 120$$

From these values of the unknown function we obtain the following conclusions

$f(n)=\log_n(n!),\ n>1.$

Now let's test the accuracy of this formula using mathematical induction.

First for $n=2$ we define $f(2)=\log_2(2!)$. And now for $n=n+1$ will be

$(n+1)^{f(n+1)-1}=n^{\log_n(n!)}=n!$

equality we get such equality

$$f(n+1)=\log_{n+1}(n!)+1=\log_{n+1}((n+1)!).$$

Thus the solution of this equation is $n>1,\ f(1)=a$ for $f(n)=\log_n(n!)$

so far it looks like *a* real number here.

In preparing students for the Olympics in all types of lifelong learning systems, it is advisable to find out what topics the student is struggling with and have an independent knowledge of the subject. These classes help students develop independent thinking skills, develop self-confidence, increase interest in the study of natural Sciences and develop creative thinking skills. These aspects will help students develop the skills they need to organize their future activities.

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