CALCULATION OF THE CONTOUR PRESSURE DISTRIBUTION IN THE OILFIELD IN CYLINDRICAL COORDINATE SYSTEM

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Abstract: *this article describes a mathematical model of an oil field in a cylindrical coordinate system, taking into account the planned time to put the field into operation. Article displays the results of numerical calculations of the reservoir pressure and production rate's values for the resulting model. The formulation of the difference problem is performed to calculate the values of bottomhole pressure and contour pressure (for approximation the Duhamel integral, the right rectangles method is used, the Thomas method is used to solve the difference problem), in addition, calculated the values of production rate function during putting the oil field into operation.*
Keywords: *oil modeling, reservoir pressure, numerical modeling, cylindrical coordinates, exploitation.*

ВЫЧИСЛЕНИЕ РАСПРЕДЕЛЕНИЯ КОНТУРНОГО ДАВЛЕНИЯ В НЕФТЯНОМ МЕСТОРОЖДЕНИИ В ЦИЛИНДРИЧЕСКОЙ СИСТЕМЕ КООРДИНАТ

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Аннотация: *в данной статье описывается математическая модель нефтяного месторождения в цилиндрической системе координат с учетом планируемого времени ввода месторождения в эксплуатацию, отображены результаты численных вычислений значений пластового давления и дебита для полученной модели. Производится постановка разностной задачи для вычисления значений забойного и контурного давления (для аппроксимации интеграла Дюамеля используется метод правых прямоугольников, для решения разностной задачи используется метод Томаса), а также функции дебита в процессе ввода*

месторождения в эксплуатацию. Получена разностная форма поставленной задачи, а также произведена компьютерная реализация разностной задачи, результаты которой отображены в данной статье.
Ключевые слова: *моделирование нефти, давление пласта, численное моделирование, цилиндрические координаты, эксплуатация.*

Introduction

Reservoir pressure is one of the important indicators for oil field development assessment. During the stage of modeling, calculation of contour and bottomhole pressures is significant stage, after which can be estimated the productivity of well and analyzed the optimal regime of oilfield development. For example, to get information about recommended diameters or length of pipes, necessary to calculate reservoir pressure and saturation pressure, which we can take with the help of mathematical modeling and numerical calculations.

In practice, when a well production is started, the pressure (in comparison with the reservoir's pressure) decreases. In the area of the bottom of the well should be observed areas of reduced pressure. The beginning of the production is made with a constant value of production rate until the establishment of the optimal pressure.

Problem statement

The contour of the oil field has a shape close to the circumference.

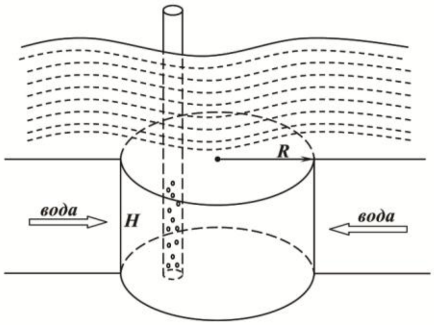

Fig. 1. Oil field contour

It is defined that (r, φ) – nonlinear coordinate system:

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$z = z$$

To consider the process, lets define the region (x, y, z, t) and write the piezoconductivity equation in the given region: (1)

$$\frac{\partial P}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\chi(r) r \frac{\partial P}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\chi(r) r \frac{\partial P}{\partial \varphi}\right) + \frac{\partial^2 P}{\partial z^2} \qquad (1)$$

where χ(r) – piezoconductivity indicator for the oil, $\frac{m^2}{s}$

$$\chi(r) = \begin{cases} \chi_1 = \frac{k_1}{\mu_0 \beta} & r_c \le r \le h \\ \chi_2 = \frac{k_2}{\mu_0 \beta} & h < r \le R \end{cases} \qquad (2)$$

Where μ – oil viscosity.

The formula (2) is called the Carslow-Jaeger formula, which was obtained in 1964[1].

β – coefficient of viscoelasticity of an oil formation, $\frac{1}{\text{Pa}}$.

Equation (1) can be simplified by considering separately the plane-parallel and axial-symmetric motion of the liquid.

Let's put forward the hypothesis of symmetric-radial motion (towards the well $\frac{\partial P}{\partial \varphi} = 0$):

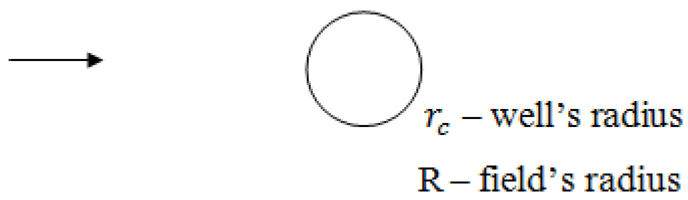

Fig. ***Ошибка! Текст указанного стиля в документе отсутствует.****2. Symmetric-radial motion*

Suppose that the motion is homogenius. It should be noted that the well can be perfect ($\frac{\partial P}{\partial z} = 0$) and non-perfect ($\frac{\partial P}{\partial z} \neq 0$) . For a perfect well $\frac{\partial P}{\partial z} = 0$ plane-parallel motion is a homogenius motion in one plane. To describe the filtration process in an anisotropic oil formation, the Darcy law is applied.

The filtration rate is directly proportional to the pressure gradient in the porous medium and inversely proportional to the dynamic viscosity of the filter gas or liquid. In this law, the ability of a rock to flow liquids and gases is characterized by a coefficient of proportionality k. In the SI system, the permeability coefficient is measured in [m^2]; In the GHS system - in [cm^2]; In the system of NPG (oilfield geology) - in [D] (Darcy):1 D = 1,02·10–8 cm^2= 1,02 10–12 m^2= 1,02 mcm^2 ≈ 1 mcm^2. [2]

$$\vec{v} = \frac{k}{\mu} s \nabla P \qquad (3)$$

where: k - permeability tensor, m^2;

μ – viscosity coefficient, Pa*s;

For water $\mu_w = 1\ mPa * s$ is acceptable,

For oil take $\mu_O = 4.5\ mPa * s,$

$k = 0{,}1 * 10^{-12} m^2$

Units of measurement for the formula (3): $\frac{m^2}{Pa*s} m^2 \frac{Pa}{m} = \frac{m^3}{s}$

Let's advance one more hypothesis: $P(r, \varphi_1) = P(r, \varphi_2)$, $P_1 \neq P_2$.

Taking into account all the points determined above, we formulate a one-dimensional equation (4):

$$\frac{\partial P}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\chi(r) r \frac{\partial P}{\partial r}) \qquad (4)$$

To construct a mathematical model and its solution, we define initial boundary conditions and construct a boundary value problem.

At the initial instant of time (t = 0), the required function P(r, t) takes the form P(r):

$$P|_{t=0} = P_0(r) \qquad (5)$$

We also define the conditions on boundaries of the domain:

$$P|_{r=R} = P_c(t) \qquad (6)$$

$$\frac{k}{\mu} s \frac{\partial P}{\partial r}|_{r=r_c} = Q(t) \qquad (7)$$

Where:

$P_c(t)$ - contour pressure;

S –surface of the well;

Q (t) - production rate of the well.

The problem (4) - (7) is called the boundary value problem.

To calculate the production rate Q (t) of an oil well, I will introduce an additional condition, since both the production rate and other key indicators of oil well productivity depend on the time of putting the well into operation. Denote t* - the time of putting the well into operation. Now, considering this index, we will write the equations for calculating the production rate: (8)-(9)

$$Q(t) = \begin{cases} \alpha f(t) & , \quad 0 \leq t \leq t^* \qquad (8) \\ \alpha f(t^*) = Q_{max} & , \quad t^* < t \leq t_{\max} \qquad (9) \end{cases}$$

Where α is the rate of oil extraction, f (t) is some function that depends on the time t.

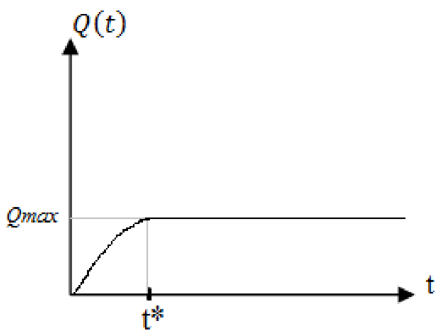

Fig. ***Ошибка! Текст указанного стиля в документе отсутствует.****3. Production rate*

For the problem solution, it would be more reasonable to find a pressure by the scheme depending an a variable step, so let's determine this step by the following function:

$$u = ln\frac{r}{r_w}, \quad u_{max} = ln\frac{r_c}{r_w} \qquad (10)$$

Thus, the function of pressure depends on u and t: $P(u, t)$

And the initial mathematical model transforms to:

$$\frac{\partial P}{\partial t} = \frac{1}{{r_c}^2 e^{2u}}\frac{\partial}{\partial u}\left(\chi(r_c e^u)\ \frac{\partial P}{\partial u}\right) \qquad (11)$$

$$P|_{t=0} = P_0(r_c e^u) \qquad (12)$$

$$\frac{k}{\mu_w}2\pi\, {r_c}^2 e^{2u}\frac{\partial P}{\partial u}|_{u=0} = Q(t) \qquad (13)$$

$$P|_{u=u_{max}} = P_c(t) \qquad (14)$$

The values of the contour pressure will be calculated from formula:

$$\Delta P_c(t) = P_0 - P_c(t) \qquad (15)$$

Where P_0 - initial contour pressure, the value of $\Delta P_c(t)$ is determined by solving the Duhamel integral: [4]

$$\Delta P_c(t) = \frac{\mu_w}{2bkh}\int_0^\tau \frac{\partial Q}{\partial \tau}\sqrt{\frac{\chi(r)(t-\tau)}{\pi}}\,d\tau \qquad (16)$$

Where b is the width of the deposit, h is the thickness of the reservoir, τ is the dimensionless time determined from the dependence: $\tau = \frac{t\chi(r)}{R^2}$, μ_w – water viscosity.

Since it was already indicated in formulas (8) - (9) that Q (t) = αf (t),

Then it is obvious that the derivative:

$$\frac{\partial Q}{\partial t} = \frac{\partial Q}{\partial \tau}\cdot\frac{\partial \tau}{\partial t} = \alpha\frac{\partial f(t)}{\partial t} = \alpha\frac{\partial f(\tau)}{\partial \tau}\cdot t' = \alpha\frac{\chi(r)}{R^2}\cdot\frac{\partial f(\tau)}{\partial \tau}$$

We introduce an additional notation to simplify the notation:

$$c(r) = \frac{\mu_w \alpha\sqrt{\chi}(r)}{2bkh\sqrt{\pi}} \qquad (17)$$

$$\Delta P_c(t) = \begin{cases} c(r)\displaystyle\int_0^\tau f'(\tau)\sqrt{(t-\tau)}\,d\tau, & 0 \le t \le t^* \quad (18) \\ c(r) f'(t^*)\sqrt{(t-\tau)}\ , & t^* < t \le t_{\max} \quad (19) \end{cases}$$

Numerical calculation and results interpretation

To calculate the Duhamel integral by formulas (18) - (19), we use the approximate method of solving the integral (rectangle method).

Right Rectangle Rule: You can approximate the exact area under a curve between a and b,$\int_a^b f(x)dx$ with a sum of right rectangles given by this formula:

$$R_n = \frac{b-a}{n}(f(x_1) + f(x_2) + \cdots + f(x_n)) \qquad (20)$$

Where, n is the number of rectangles, $\frac{b-a}{n}$ is the width of each rectangle, and the function values are the heights of the rectangles, and it is shown on the Figure 4 [3].

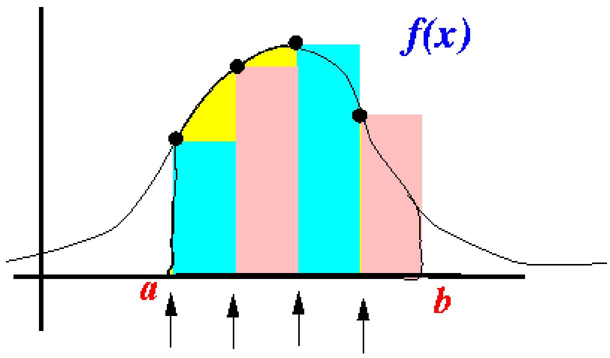

Fig. 4. Rectangles method

Then for the numerical solution we construct a grid:

j = 0, 1, … , m, … , n; i = 0, 1, … , M, … , N;

It means that the segment (0, t^*) is divided into m equal parts, but the segment (0, t_{max}) is divided into n equal parts, and similar division is for the segment (0; u_{max}). Steps by the time Δt by the coordinate Δu are given, then: $m = \frac{t^*}{\Delta t} = \frac{t_{max}}{\Delta t}$;

$$N = \frac{u_{max}}{\Delta u} \qquad (21)$$

Thus, time iterated items:

$$t_j = j\Delta t; u_i = i\Delta u \qquad (22)$$

$$\Delta P_c(t) = c(r)\int_0^\tau f'(\tau)\sqrt{(t-\tau)}\,d\tau \approx c(r)\Delta\sum_{j=0}^{m}\frac{\partial f(\tau_{j+0.5})}{\partial\tau}\sqrt{(t_{j+1}-\tau_{j+0.5})} =$$
$$= \Delta P_c[m]\tau\,, \quad 0 \le \tau \le \tau^* \qquad (23)$$

$$\Delta P_c(t) = c(r)f'(t^*)\sqrt{(t-\tau)} \approx \Delta P_c[m] + c(r)Q_{max}\sqrt{(t_{j+1}-\tau^*)},$$
$$\tau^* < \tau \le \tau_{\max} \qquad (24)$$

In the same way, numerically, all the values of the contour pressure at each point are considered, in the form:

$$P_c(t_j) = P_0 - \Delta P_c(t_j) \qquad (25)$$

The figure 5 demonstrates how the production rate changes. It reaches the maximum value and becomes constant at the point t^*. It means that during well's commissioning time the "efficiency"of the well increases. The growth in values of Q has a big significance in practice.

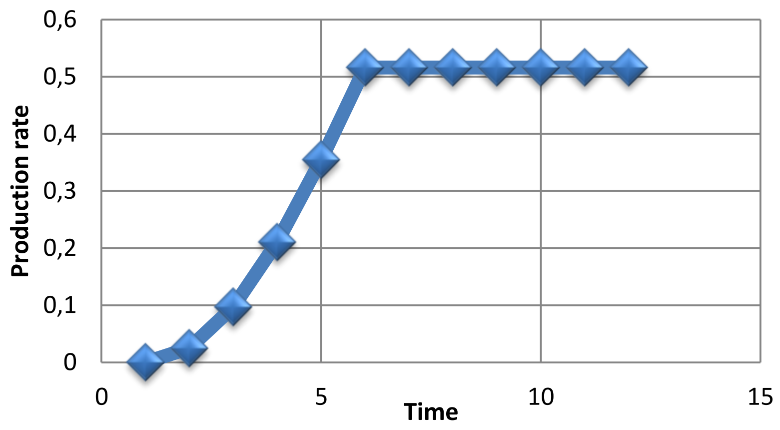

Fig. 5. Production rate

The Figure 6 shows the values of contour pressure.

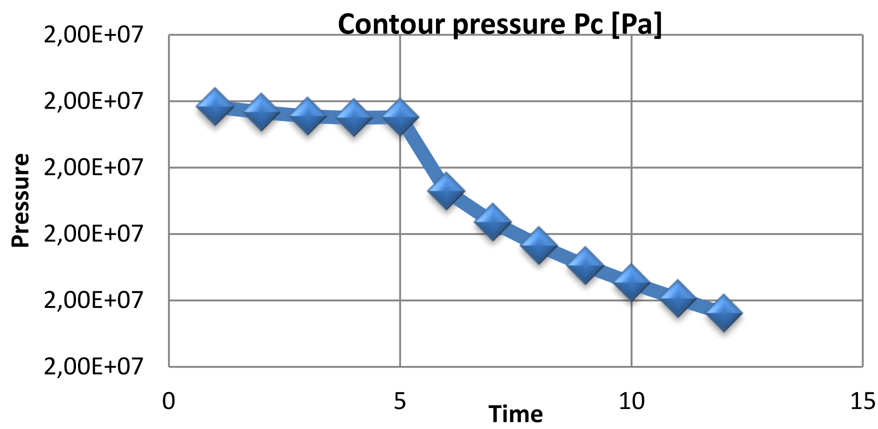

Figure 6. Contour pressure

In the Figure 7 the bottomhole pressure (Po) is shown.

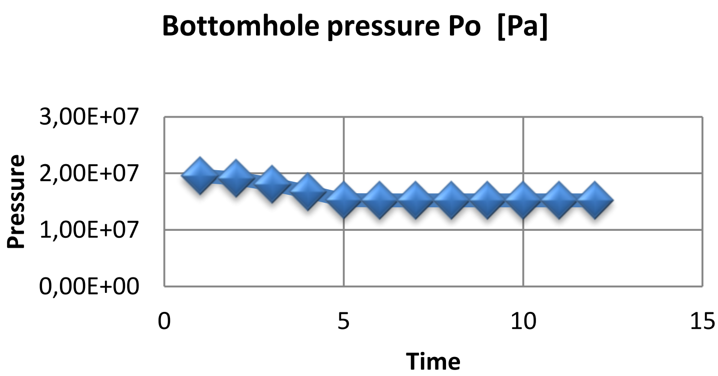

Fig. 7. Bottomhole pressure

And finally, let's determine the pressure at each point u_i, but to achieve this goal it's necessary to build a difference scheme for the model (11) - (14):

$$r_w{}^2 e^{2u} \frac{Y_i^{j+1} - Y_i^j}{\Delta t} = \frac{1}{\Delta u}\left(\chi_{i+0.5} \frac{Y_{i+1}^{j+1} - Y_i^{j+1}}{\Delta u} - \chi_{i-0.5} \frac{Y_i^{j+1} - Y_{i-1}^{j+1}}{\Delta u}\right), i = 1, \ldots, N - 1; j = 0, \ldots, M - 1 \quad (26)$$

$$Y_i^0 = P_0(r_w e^u), \ i = 0, \ldots, N \qquad (27)$$

$$\frac{k}{\mu_w} 2\pi\, r_w{}^2 e^{2u} \frac{Y_1^{j+1} - Y_0^{j+1}}{\Delta u} = Q(t), \ j = 0, \ldots, M - 1 \qquad (28)$$

$$Y_N^{j+1} = P_c(t), \ j = 0, \ldots, M - 1 \qquad (29)$$

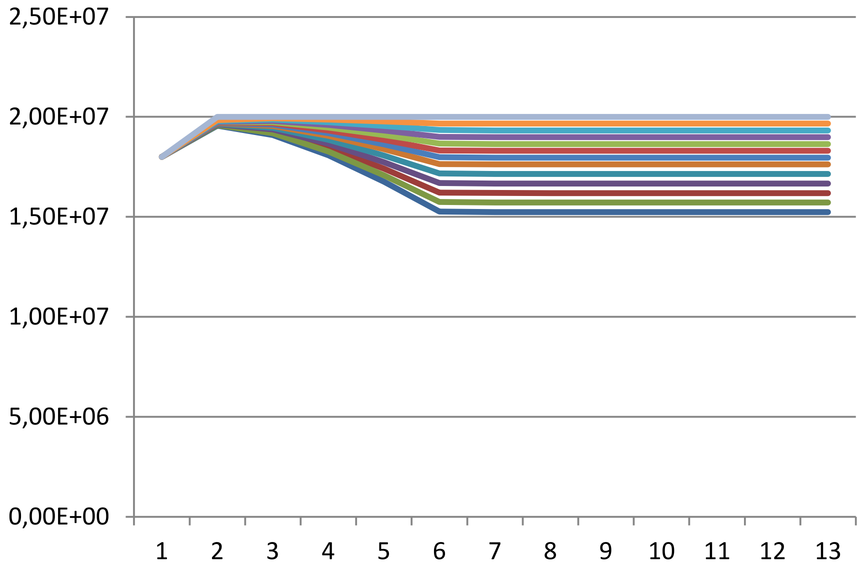

Fig. 8. Pressure (inside the grid)

Conclusion

After work performed during the paper's writing, it was conducted that in oil well development process, the values obtained using the approximation of a difference problem have a practical significance. The algorithm for model solution was successfully implemented in the Java programming language. The method of solving this problem can be used for other differential problems in oil fields modeling sphere.

Список литературы / References

1. *Karslow G., Eger D.* The heat conductivity of solid bodies. M.: Nauka, 1964. 488 p.
2. *Dmitriev N.M., Kadet V.V.* Vvedenie v podzemnuyu gidromekhaniku. M.: Intercontact Nauka, 2003. 250 p.

3. *Sandu C.F., Roslyak A.T., Galkin V.M.* Practicum on «Razrabotka neftyanikh I gazovikh mestorozhdeniy» discipline. Tomsk: Publishing house of Tomsk Polytechnic University, 2011. 92 p.