PROBLEMS WITH PARAMETERS SOLVED WITH THE SEARCH FOR NECESSARY CONDITIONS

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Abstract: *this article deals with problems with parameters that can be solved with the search for necessary conditions. The proposed problems sufficient conditions for the desired values of the parameter were established as a result of the verification and it was shown that by successively increasing the necessary condition, it can be reduced to a sufficient one. To solve the equation* $F(x)=G(x)$ *. First, the entire set of real numbers is seen in the fact that they are the roots of the equation. We exclude those that have an obvious justification: the roots of our equation can not meet outside the domains of the definition of functions* $F(x)$ *and* $G(x)$*.*

Keywords: *problem, function, roots of the equation, equality.*

ПРОБЛЕМЫ С ПАРАМЕТРАМИ, РЕШАЕМЫМИ С ПОИСКОМ НЕОБХОДИМЫХ УСЛОВИЙ

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Аннотация: *в данной статье рассматриваются проблемы с параметрами, решаемыми с поиском необходимых условий. В предлагаемых задачах достаточные условия для искомых значений параметра устанавливались в результате проверки и показано, что последовательным усилением необходимого условия можно свести его к достаточному. Как решить это уравнение* $F(x)=G(x)$*. Во-первых, весь набор действительных чисел рассматривается в том, что они являются корнями уравнения. Мы исключаем те, которые имеют очевидное оправдание: корни нашего уравнения не могут встречаться вне областей определения функции* $F(x)$ *и* $G(x)$*.*

Ключевые слова: *задача, функция, корни уравнения, равенство.*

Let it is necessary to solve the equation $F(x)=G(x)$. First, the entire set of real numbers is seen in the fact that they are the roots of the equation. We exclude those that have an obvious justification: the roots of our equation can not meet outside the domains of the definition of functions $F(x)$ and $G(x)$. If it was possible to establish that all the roots of the equation lie in a cetain set *M*, there is thus a necessary condition under which equality $F(x)=G(x)$: If x is a root of the equation, then $x\in M$. We shall call the problems solved by this method, the problems with the search for necessary conditions. Let us illustrate what was said on specific examples.

Problem 1. For what values of parameter *A* does the equation

$$x^2-2A\sin(\cos x)+2=0$$

have *a* unique solution? [2]

Solution. Obviously, $x=0$ is a necessary condition for the existence of a single root of the given equation. For $x=0$ we have

$$A=\frac{1}{\sin 1}$$

We prove that this parameter value the desired. In fact, when $A=\frac{1}{\sin 1}$ we obtain

$$x^2+2=\frac{2}{\sin 1}\sin(\cos x) \qquad (1)$$

It prove that this, $x^2+2\ge 2$. We consider the function $f(t)=\sin t$ $t\in[-1;1]\subset\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ the function $f(t)$ increases. Consequently, $\sin(\cos x)\le\sin 1$.

From here

$$\frac{2\sin(\cos x)}{\sin 1}\le\frac{2\sin 1}{\sin}=2$$

Hence, equation (1) is equivalent to a system

$$\begin{cases} x^2+2=2 \\ \dfrac{2\sin(\cos x)}{\sin 1}=2 \end{cases}$$

having a unique solution $x=0$.

answer $A=\frac{1}{\sin 1}$

Problem 2. Find all *A* such that for any *B* the system [1]

$$\begin{cases} \left(1+3x^2\right)^A+\left(B^2-4B+5\right)^z=2 \\ x^2y^2-(2-B)xy+A^2+2A=3 \end{cases}$$

has at least one solution.

Solution. This system must have at least one solution for any *B*, and hence for $B=2$.

In this case

$$\begin{cases} \left(1+3x^2\right)^A=1 \\ x^2y^2+A^2+2A=3 \end{cases}$$

Whence

$$\begin{cases} x=0 \\ A^2+2A=3 \end{cases} \quad \text{or} \quad \begin{cases} A=0 \\ x^2y^2=3 \end{cases}$$

The circle of suspect values for variable *A* narrowed to a multitude $\{-3;0;1\}$.

For $A=-3$ we obtain

$$\begin{cases} \dfrac{1}{\left(1+3x^2\right)^3}+\left(B^2-4B+5\right)^y=2 \\ x^2y^2-(2-B)xy=3 \end{cases}$$

If $B\ne -2$, then the solution of the first equation $y=0$ does not satisfy the second equation, but by hypothesis the system must have a solution for any *B*, so $A=0$ will have to be excluded from the list of.

At $A=1$

$$\begin{cases} 3x^2+\left(B^2-4B+5\right)^y=1 \\ x^2y^2-(2-B)xy=0 \end{cases} \qquad (2)$$

have a unique solution.

Solution. It is obvious that if (x_0,y_0) solution system (2), then $(-x_0,y_0)$ is also its solution. Therefore condition $x=0$ necessary for the existence our system can have a few solutions kind of $(-x_0,y_0)$ or do not have solutions. We set, $x=0$. Then

$$\begin{cases} a = y+1 \\ y^2 = 1 \end{cases}$$

from here $a = 0$ or $a = 2$.

Thus, the desired value of the parameter must be chosen in the set $\{0; 2\}$. When $a = 0$ is obtained

$$\begin{cases} y+1-|x| = 0, \\ x^2 + y^2 = 1, \end{cases}$$

location

$$\begin{cases} |x| = y+1 \\ y^2 + 2y + 1 + y^2 = 1, \end{cases}$$

Therefore

$$\begin{cases} x = 1 \\ y = 0 \end{cases} \quad \text{or} \quad \begin{cases} x = -1 \\ y = 0 \end{cases} \quad \text{or} \quad \begin{cases} x = 0 \\ y = -1 \end{cases}$$

As you can see, there were three solutions of the system, so $a = 0$ does not satisfy the condition of the problem.

At $a = 2$ we get:

$$\begin{cases} 2x^4 + |x| = y-1, \\ x^2 + y^2 = 1 \end{cases}$$

Obviously, $2x^4 + |x| \geq 0$. From the first equation we have $y \geq 1$, from the second $y \leq 1$. Hence, $y = 1$, hence $x = 0$. The verification shows that the pair is a $(0,1)$ - solution, and, in view of the restriction to the variable y $(y \geq 0$ and $y \leq 1)$ it is unique.

Answer 2

Problem 4. For any negative integer a, the function $F(x) = \cos ax \sin \frac{15x}{a^2}$ satisfies condition $F(x+5\pi) = F(x)$ for all $x \in R$.

Solution. By the condition of the problem, equality $\cos(x+5\pi)\sin\frac{15(x+5\pi)}{a^2} = \cos ax \sin\frac{15x}{a^2}$ must be satisfied for any real x, and hence also for $x = 0$.

In this case we have

$$\cos 5\pi a \cdot \sin\frac{75\pi}{a^2} = 0$$

since $\cos 5\pi a \neq 0$, then $\sin\frac{75\pi}{a^2} = 0$, those $\frac{75}{a^2} = k$, where *k* is an integer.

Hence $a^2 = 1$ or $a^2 = 25$. We are only interested in negative values of *a*, so the solution of the problem should be sought in the set $\{-1; -5\}$. The verification shows that for both $a = -1$ and $a = -5$ our equality holds for all real *x*.

ans: $a = -1$ or $a = -5$.

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