Kinematic analysis of a new harrow type wool transport mechanism

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Кинематический анализ нового типа боронного механизма в шерстомоечной машине

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Abstract: *this paper presents the scientific results of the kinematic calculation and the law of motion of a new wool transport mechanism to reduce fiber entanglement.*

Аннотация: *в этой статье даны научные результаты кинематического расчёта и закона движения боронного механизма в новой шерстомоечной машине с учётом снижения запутанности шерстяного волокна.*

Keywords: *four bar linkage, positions analysis, velocity analysis, acceleration analysis, wool transport mechanism, kinematic analysis*

Ключевые слова: *четырёхзвенный механизм, анализ позиций, анализ скоростей, анализ ускорения, боронный механизм, кинематический анализ.*

Synthesis of a new harrow type wool transport mechanism has presented in [1]. This paper dedicated for the kinematic analysis of the proposed wool transport mechanism.

The four bar of the proposed mechanism shown in Figure 1. The crank O_1A is fixed on pivot O_1 that is located at the origin of the coordinate system. The crank rotates counterclockwise with θ_2 angle about fixed pivot O_1. The point A of the coupler ABC rotates follows to the crank rotation. The rocker or lever O_2B move in oscillation motion with respect to the coupler motion. The output link of the mechanism is the harrow and it connects with its one end to the point C. The desired motion of the mechanism has been obtained by point C.

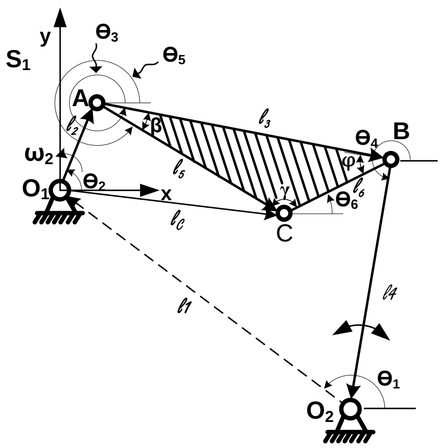

Figure 1. Vector representation of the four bar linkage [2]

1. Position analysis

The position analysis of the four bar linkage starts from defining the coordinates of centres of the revolute joints in a coordinate system. A reference system S1 (*x, y, O1*) with ***x*** axis or horizontal and the origin of the system at point O1, centre of the revolute joint connecting links 1 and 2 are chosen. Coordinates of the pivots O_1 and O_2 can be written as follows:

$$O_1 = [x_{O_1}\, y_{O_1}] = [0\ 0]; \tag{1}$$

$$O_2 = [x_{O_2}\, y_{O_2}] = O_1 - l1[cos\theta_1\ sin\theta_1]; \tag{2}$$

The point A rotates about the origin of the coordinate system S1; its coordinates can be written as:

$$A = [x_A\, y_A] = O_1 + l_2[\ cos\theta_2\ sin\theta_2]; \tag{3}$$

Coordinates of point B:

Referring to the Figure 10, if the coordinates of the points A and O_2 are known, the coordinates of the point B can be found by using the equation for circle as follows:

$$\left.\begin{matrix} (x_B - x_{O_2})^2 + (y_B - y_{O_2})^2 = l_4^2 \\ (x_B - x_A)^2 + (y_B - y_A)^2 = l_3^2 \end{matrix}\right\} B(x_B\, y_B); \tag{4}$$

The Equation (4) should be developed, simplified and one subtracted to the other as follows:

$$\left.\begin{matrix} x_B^2 - 2x_B x_{O_2} + x_{O_2}^2 + y_B^2 - 2y_B y_{O_2} + y_{O_2}^2 - l_4^2 = 0 \\ x_B^2 - 2x_B x_A + x_A^2 + y_B^2 - 2y_B y_A + y_A^2 - l_3^2 = 0 \end{matrix}\right\} \tag{5}$$

$$-2x(x_{O_2} - x_A) + x_{O_2}^2 - x_A^2 - 2y(y_{O_2} - y_A) + y_{O_2}^2 - l_4^2 - y_A^2 + l_3^2 = 0; \tag{6}$$

Here is $x_B = x;\ y_B = y;$

$$x + y\frac{y_{O_2} - y_A}{x_{O_2} - x_A} = \frac{x_{O_2}^2 - x_A^2 + y_{O_2}^2 - y_A^2 + l_3^2 - l_4^2}{2(x_{O_2} - x_A)} = \frac{N(O_2)^2 - N(A)^2 + l_3^2 - l_4^2}{2(x_{O_2} - x_A)}; \tag{7}$$

The Equation (7) can be simplified as:

$$\frac{y_{O_2} - y_A}{x_{O_2} - x_A} = A \tag{8}$$

$$\frac{N(O_2)^2 - N(A)^2 + l_3^2 - l_4^2}{2(x_{O_2} - x_A)} = B \tag{9}$$

The following simple equation can be obtained for $\boldsymbol{x}$:

$$x = -Ay + B; \tag{10}$$

The Equation (10) can be inserted into the first line of the Equation system (5) and can be written as follows:

$$A^2y^2 + B^2 - 2ABy + 2x_{O_2}Ay - 2x_{O_2}B + x_{O_2}^2 + y^2 - 2yy_{O_2} + y_{O_2}^2 - l_4^2 = 0; \tag{11}$$

$$(A^2 + 1)y^2 + (2x_{O_2}A - 2AB - 2y_{O_2})y + (B^2 - 2x_{O_2}B + N(O_2)^2 - l_4^2) = 0; \tag{12}$$

$$a = (A^2 + 1); \tag{13}$$

$$b = (2x_{O_2}A - 2AB - 2y_{O_2}); \tag{14}$$

$$c = B^2 - 2x_{O_2}B + N(O_2)^2 - l_4^2; \tag{15}$$

$$ay^2 + 2by + c = 0; \tag{16}$$

$$y = \frac{-b \pm \sqrt{b^2 - ac}}{a}; \tag{17}$$

Coordinates of point C can be found as:

$$\overline{O_1C} = \overline{O_1A} + \overline{AC};$$

$$C = l_2(cos\theta_2 sin\theta_2) + l_5(\cos(\theta_3 - \beta)\ \sin(\theta_3 - \beta));$$

$$\left.\begin{matrix} X_C = l_2cos\theta_2 + l_5cos(\theta_3 - \beta) \\ Y_C = l_2sin\theta_2 + l_5sin(\theta_3 - \beta) \end{matrix}\right\} \tag{18}$$

Here is, $\beta = cos^{-1}\left[\frac{l_5^2 + l_3^2 - l_6^2}{2l_5l_3}\right];$ (19)

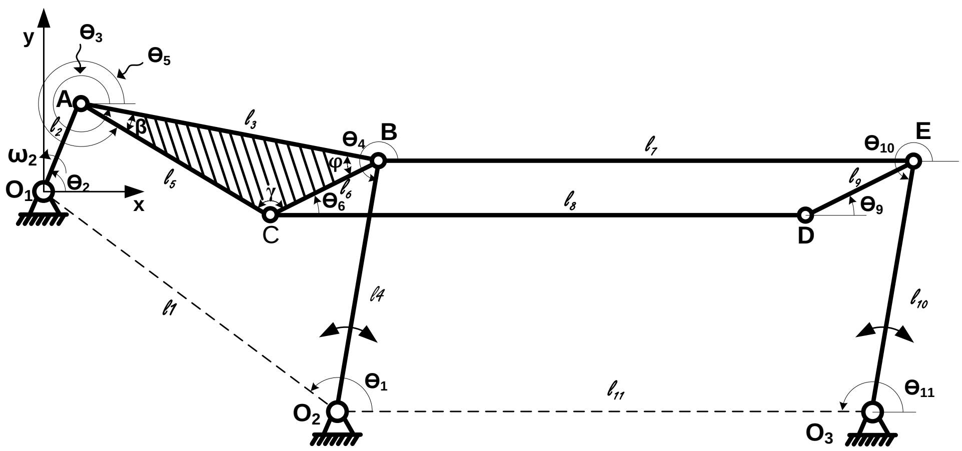

Figure 2. The position analysis of the proposed mechanism [2]

Coordinates of the pivot O_3 can be found as follow:

$O_3 = [x_{O_3}\ y_{O_3}] = O_2 + l_{11}[cos\theta_{11}\ \ \sin\theta_{11}];$ (20)

Coordinates of the point E:

$E = B + l_7[\cos\theta_7\ \ sin\theta_7];$ (21)

Coordinates of the point D:

$D = C + l_8[\cos\theta_8\ \ sin\theta_8];$ (22)

Loop closure equations for four bar linkage can be written as follow:

$\boldsymbol{l_2} + \boldsymbol{l_3} + \boldsymbol{l_4} + \boldsymbol{l_1} = 0;$ (23)

Rewriting the Equation (23) in its ***x*** and ***y*** axis component equations:

$l_2cos\theta_2 + l_3cos\theta_3 + l_4cos\theta_4 + l_1cos\theta_1 = 0;$ (24)

$l_2sin\theta_2 + l_3sin\theta_3 + l_4sin\theta_4 + l_1sin\theta_1 = 0;$ (25)

We know that θ_2 is known, and θ_1 is also known, constant. In order to eliminate θ_3, we first isolate it on one side of the Equations (24) and (25):

$l_3cos\theta_3 = -l_1cos\theta_1 - l_4cos\theta_4 - l_2cos\theta_2;$ (26)

$l_3sin\theta_3 = -l_1sin\theta_1 - l_4sin\theta_4 - l_2sin\theta_2;$ (27)

Both sides of the equations (26) and (27) should be squared, added and the result simplified using the trigonometric identity $sin^2\theta + cos^2\theta = 1;$

This gives:

$l_3^2 = l_1^2 + l_2^2 + l_4^2 + 2l_1l_4(cos\theta_1cos\theta_4 + sin\theta_1sin\theta_4) -$

$-2l_1l_2(cos\theta_1cos\theta_2 + sin\theta_1sin\theta_2) - 2l_2l_4(cos\theta_2cos\theta_4 + sin\theta_2sin\theta_4);$ (28)

The Equation (28) gives θ_4 in terms of the given angle θ_2 (and constant angleθ_1), but not explicitly. To obtain explicit expression, the Equation (28) should be simplified by combining the coefficients of $cos\theta_4$ and $sin\theta_4$ as follows:

$Acos\theta_4 + Bsin\theta_4 + C = 0;$ (29)

Where,

$$\left.\begin{array}{r} \mathrm{A} = 2\mathrm{l}_1\mathrm{l}_4\cos\theta_1 - 2\mathrm{l}_2\mathrm{l}_4\cos\theta_2; \\ \mathrm{B} = 2\mathrm{l}_1\mathrm{l}_4\sin\theta_1 - 2\mathrm{l}_2\mathrm{l}_4\sin\theta_2; \\ \mathrm{C} = \mathrm{l}_1^2 + \mathrm{l}_2^2 + \mathrm{l}_4^2 - \mathrm{l}_3^2 - 2\mathrm{l}_1\mathrm{l}_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \end{array}\right\}; \quad (30)$$

To solve the Equation (29), standard trigonometric identities can be used for half angles given in the following:

$sin\theta_4 = \frac{2\tan(\theta_4/2)}{1+tan^2(\theta_4/2)};$ (31)

$cos\theta_4 = \frac{1-tan^2(\theta_4/2)}{1+tan^2(\theta_4/2)};$ (32)

After substitution and simplification, the following equation can be obtained:

$(C - A)t^2 + 2Bt + (A + C) = 0;$ (33)

Where, $t = \tan\left(\frac{\theta_4}{2}\right);$

Solving for *t* gives:

$t = \frac{-2B+\sigma\sqrt{4B^2-4(C-A)(C+A)}}{2(C-A)} = \frac{-B+\sqrt{B^2-C^2+A^2}}{C-A};$ (34)

And $\theta_4 = 2\tan^{-1} t;$ (35)

Equations (26) and (27) can now be solved for θ_3. Dividing the Equation (27) by (26) and solving for θ_3 gives:

$$\theta_3 = \tan^{-1}\left[\frac{-l_4 sin\theta_4 - l_2 sin\theta_2 - l_1 sin\theta_1}{-l_4 cos\theta_4 - l_2 cos\theta_2 - l_1 cos\theta_1}\right]; \quad (36)$$

In equation (34) it is essential that the sign of the numerator and the denominator be maintained to determine the quadrant in which the angle θ_3 lies. This can be done directly by using ATAN2 function. The form of this function is:

$$ATAN2(\sin\theta_3, \cos\theta_3) = \tan^{-1}\left[\frac{sin\theta_3}{cos\theta_3}\right]; \quad (37)$$

Equations (35) - (37) give a complete and consistent solution to the position problem of the four bar linkage. For any values of $\boldsymbol{\theta_2}$, there are typically two values of $\boldsymbol{\theta_3}$ and $\boldsymbol{\theta_4}$, given the substituting $\sigma = +1$ and -1, respectively.

2. Velocity analysis

The velocity equations can be developed by differentiating the Equation (23) as:

$$\dot{\boldsymbol{l}}_2 + \dot{\boldsymbol{l}}_3 + \dot{\boldsymbol{l}}_4 + \dot{\boldsymbol{l}}_1 = 0; \quad (38)$$

Rewriting the Equations (38) in its $\boldsymbol{x}$ and $\boldsymbol{y}$ axis component is the same as that differentiating the Equations (23) and (24). The resulting equations are:

$$-l_2 sin\theta_2\omega_2 - l_3 sin\theta_3\omega_3 - l_4 sin\theta_4\omega_4 - l_1 sin\theta_1\omega_1 = 0; \quad (39)$$

$$l_2 cos\theta_2\omega_2 + l_3 cos\theta_3\omega_3 + l_4 cos\theta_4\omega_4 + l_1 cos\theta_1\omega_1 = 0; \quad (40)$$

Since, θ_1 is constant, the angular velocity of link 2 ω_2 is known, the only new unknowns are ω_3 and ω_4 which are the angular velocities of the link 3 and the link 4 respectively. In matrix form, Equations (39) and (40) can be rearranged and rewritten as:

$$\begin{bmatrix} -l_3 sin\theta_3 & l_4 sin\theta_4 \\ l_3 cos\theta_3 & -l_4 cos\theta_4 \end{bmatrix} \begin{Bmatrix} \omega_3 \\ \omega_4 \end{Bmatrix} = \begin{Bmatrix} l_2 sin\theta_2\omega_2 \\ -l_2 cos\theta_2\omega_2 \end{Bmatrix}; \quad (41)$$

Solving these two equations in two unknowns yield:

$$\omega_4 = -\frac{l_2 sin\theta_2\omega_2 - \frac{l_3 cos\theta_3 \cdot l_2 sin\theta_2\omega_2}{l_3 sin\theta_3}}{l_4 sin\theta_4 - \frac{l_3 cos\theta_3 \cdot l_4 sin\theta_4}{l_3 sin\theta_3}}; \quad (42)$$

$$\omega_3 = -\frac{l_2 sin\theta_2\omega_2 + l_4 sin\theta_4 \cdot \frac{l_2 sin\theta_2\omega_2 - \frac{l_3 cos\theta_3 \cdot l_2 sin\theta_2\omega_2}{l_3 sin\theta_3}}{l_4 sin\theta_4 - \frac{l_3 cos\theta_3 \cdot l_4 sin\theta_4}{l_3 sin\theta_3}}}{l_3 sin\theta_3}; \quad (43)$$

Velocity equations of the coupler point C can be found by differentiating the Equation (18) as:

$$\dot{C} = l_2\omega_2(-sin\theta_2\ cos\theta_2) + l_5\omega_3(-sin(\theta_3 - \beta)\ cos(\theta_3 - \beta));$$

$$\left.\begin{aligned} \dot{X}_C &= -l_2 sin\theta_2\omega_2 - l_5 sin(\theta_3 - \beta)\omega_3 \\ \dot{Y}_C &= l_2 cos\theta_2\omega_2 + l_5 cos(\theta_3 - \beta)\omega_3 \end{aligned}\right\}; \quad (44)$$

3. Acceleration analysis

Since θ_1 is constant, the acceleration equations can be developed by differentiating the Equation (38) as:

$$\ddot{\boldsymbol{l}}_2 + \ddot{\boldsymbol{l}}_3 - \ddot{\boldsymbol{l}}_4 = 0; \quad (45)$$

Rewriting the Equation (2.45) in its $\boldsymbol{x}$ and $\boldsymbol{y}$ axis component equations is the same as that differentiating the Equations (2.39) and (2.40). The resulting component equations are:

$$-l_2 sin\theta_2\alpha_2 - l_2 cos\theta_2\omega_2^2 - l_3 sin\theta_3\alpha_3 - l_3 cos\theta_3\omega_3^2 + l_4 sin\theta_4\alpha_4 + l_4 cos\theta_4\omega_4^2 = 0; \quad (46)$$

$$l_2 cos\theta_2\alpha_2 - l_2 sin\theta_2\omega_2^2 + l_3 cos\theta_3\alpha_3 - l_3 sin\theta_3\omega_3^2 - l_4 cos\theta_4\alpha_4 + l_4 sin\theta_4\omega_4^2 = 0; \quad (47)$$

These equations can also be represented in matrix form, where the terms associated with the known crank acceleration and the quadratic velocity terms are moved to the right-hand side as:

$$\begin{bmatrix} -l_3 sin\theta_3 & l_4 sin\theta_4 \\ l_3 cos\theta_3 & -l_4 cos\theta_4 \end{bmatrix} \begin{Bmatrix} \alpha_3 \\ \alpha_4 \end{Bmatrix} = \begin{Bmatrix} l_2(sin\theta_2\alpha_2 + cos\theta_2\omega_2^2) + l_3 cos\theta_3\omega_3^2 - l_4 cos\theta_4\omega_4^2 \\ -l_2(cos\theta_2\alpha_2 - sin\theta_2\omega_2^2) + l_3 sin\theta_3\omega_3^2 - l_4 sin\theta_4\omega_4^2 \end{Bmatrix}; \quad (48)$$

Solving first raw of these equations yield two unknowns as:

$$\alpha_4 = \frac{\left(l_2\left(sin\theta_2\alpha_2 + cos\theta_2\omega_2^2\right) + l_3 cos\theta_3\omega_3^2 - l_4 cos\theta_4\omega_4^2\right) - \frac{l_3 cos\theta_3 \cdot l_2\left(cos\theta_2\alpha_2 - sin\theta_2\omega_2^2\right) + l_3 sin\theta_3\omega_3^2 - l_4 sin\theta_4\omega_4^2}{l_3 sin\theta_3}}{l_4 sin\theta_4 - \frac{l_3 cos\theta_3 \cdot l_4 sin\theta_4}{l_3 sin\theta_3}}; \quad (49)$$

$$\alpha_3 = \frac{l_2\left(sin\theta_2\alpha_2 + cos\theta_2\omega_2^2\right) + l_3 cos\theta_3\omega_3^2 - l_4 cos\theta_4\omega_4^2 - l_4 sin\theta_4 \cdot \frac{\left(l_2\left(sin\theta_2\alpha_2 + cos\theta_2\omega_2^2\right) + l_3 cos\theta_3\omega_3^2 - l_4 cos\theta_4\omega_4^2\right) - \frac{l_3 cos\theta_3 \cdot l_2\left(cos\theta_2\alpha_2 - sin\theta_2\omega_2^2\right) + l_3 sin\theta_3\omega_3^2 - l_4 sin\theta_4\omega_4^2}{l_3 sin\theta_3}}{l_4 sin\theta_4 - \frac{l_3 cos\theta_3 \cdot l_4 sin\theta_4}{l_3 sin\theta_3}}}{l_3 sin\theta_3}; \quad (50)$$

Acceleration equations of the coupler point C can be found by differentiating the Equation (44) as:

$$\left.\begin{aligned} \ddot{X}_C &= -l_2(\sin\theta_2\alpha_2 + \cos\theta_2\omega_2^2) - l_5(\sin\theta_5\,\alpha_3 + \cos\theta_5\omega_3^2) \\ \ddot{Y}_C &= l_2(\cos\theta_2\alpha_2 - \sin\theta_2\omega_2^2) + l_5(\cos\theta_5\,\alpha_3 - \sin\theta_5\omega_3^2) \end{aligned}\right\}; \quad (51)$$

4. Results

Results of the kinematic analysis are shown in Figures 3-10.

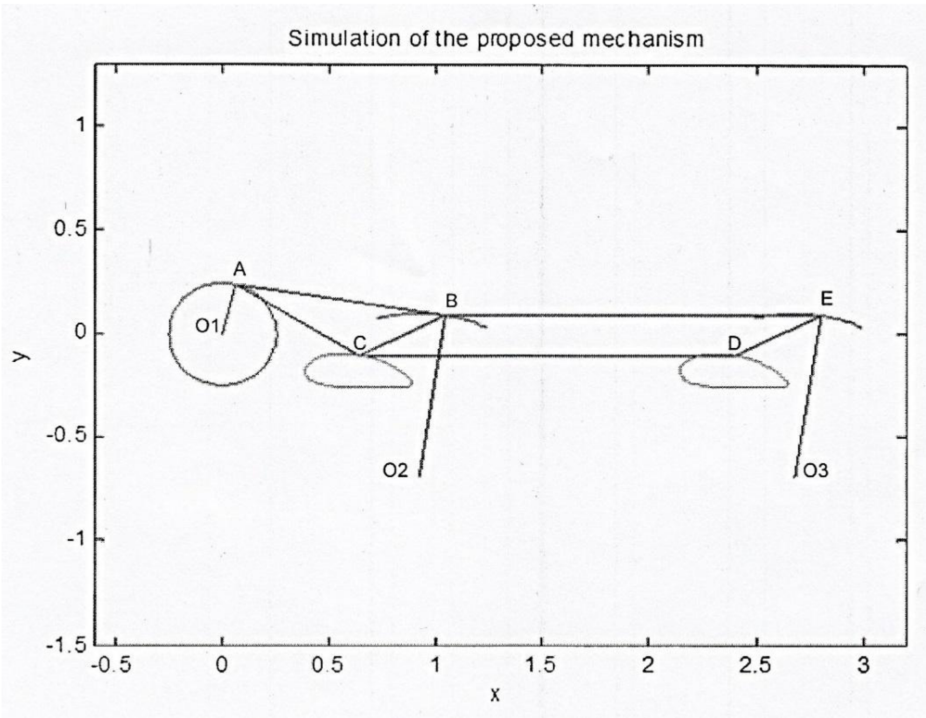

Figure 3. Simulation of the proposed mechanism

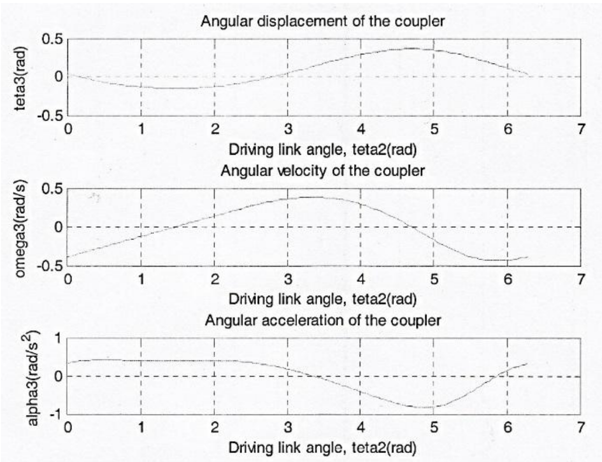

Figure 4. Angular displacement, velocity and acceleration of the coupler

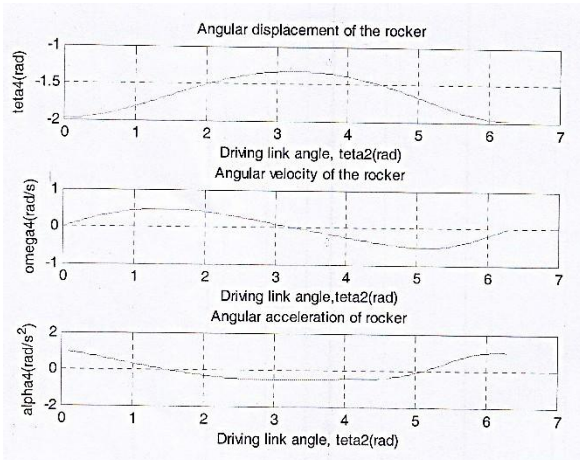

Figure 5. Angular displacement, velocity and acceleration of the rocker (link 4)

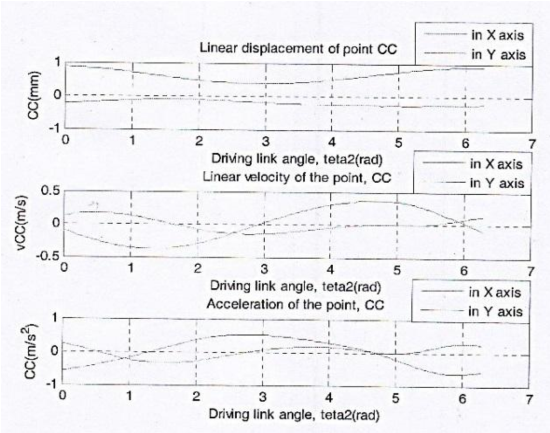

Figure 6. Linier displacement, velocity and acceleration of the point C

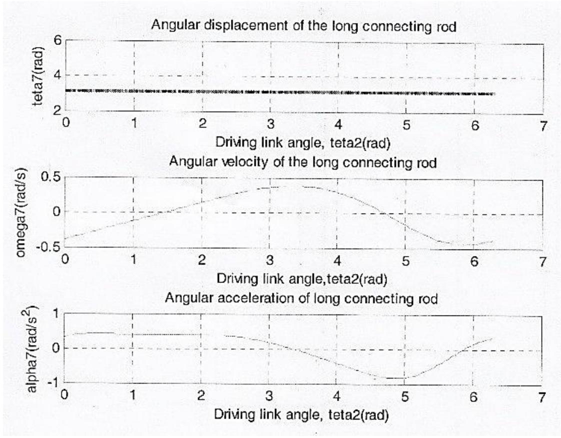

Figure 7. Angular displacement, velocity and acceleration of the long connecting rod (link 5)

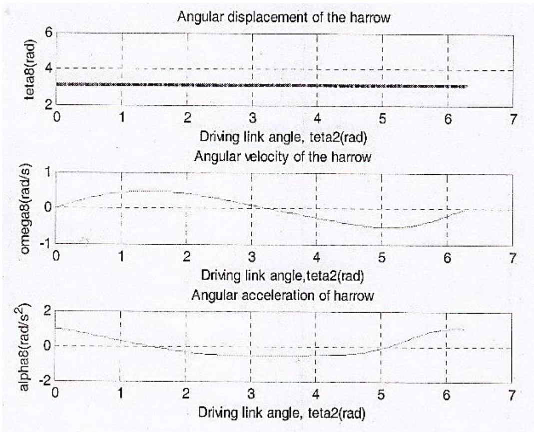

Figure 8. Angular displacement, velocity and acceleration of the harrow (link 6)

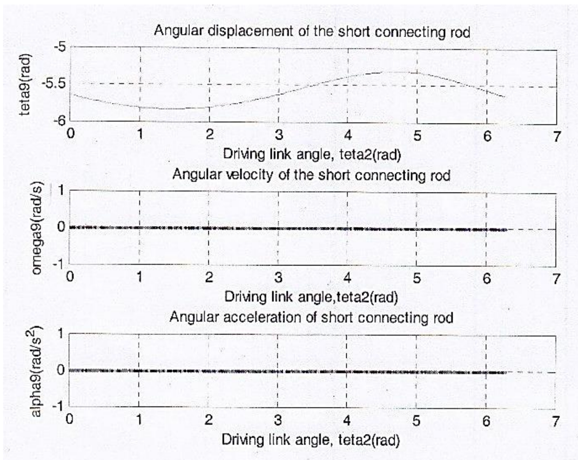

Figure 9. Angular displacement, velocity and acceleration of the short connecting rod (link 7)

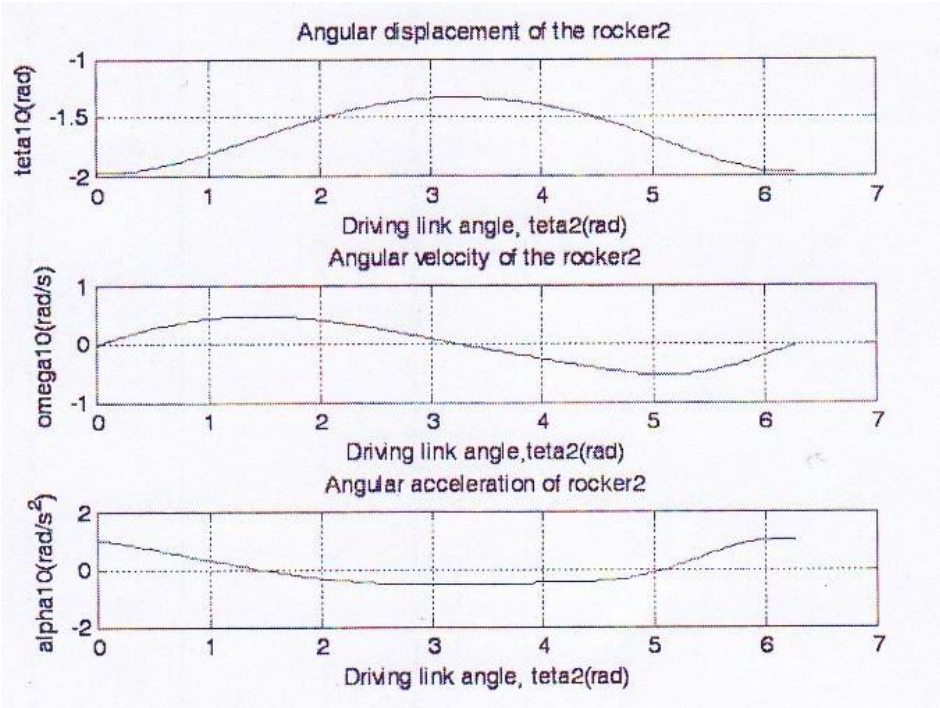

Figure 10. Angular displacement, velocity and acceleration of the rocker 2 (link 8)

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